Games for Fun and Profit

Some recent results on improved game analysis

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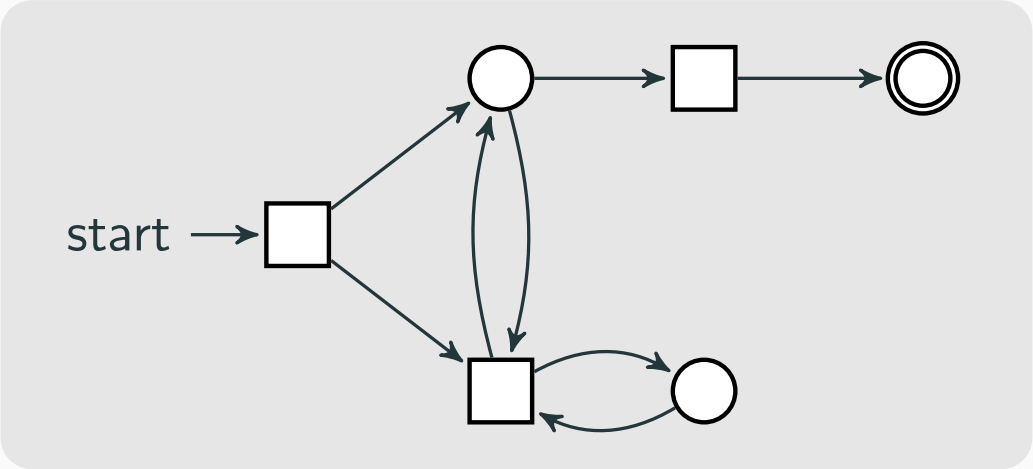
Two-Player Games

Games: algorithmic essence of verification, reasoning, synthesis, ...



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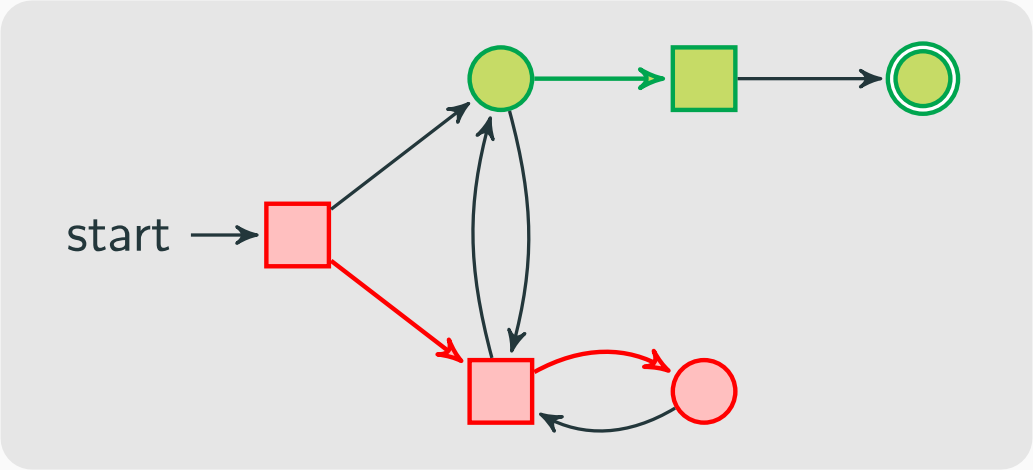

- How to compute *winning regions*?
- How to extract *winning strategies*?
- *Reduction* of problems to game solving

Some Recent Results

- COOL 2 - A Generic Reasoner for Modal Fixpoint Logics
  [CADE 2023] (O. Görlitz, M. Humml, D. Pattinson, S. Prucker, L. Schröder)

- Generic Model Checking for Modal Fixpoint Logics in COOL-MC
  [VMCAI 2024] (M. Humml, S. Prucker, L. Schröder, A. Strahlberger)

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- COOL 2 - A Generic Reasoner for Modal Fixpoint Logics [CADE 2023] (O. Görlitz, M. Humml, D. Pattinson, S. Prucker, L. Schröder)
- Generic Model Checking for Modal Fixpoint Logics in COOL-MC [VMCAI 2024] (M. Humml, S. Prucker, L. Schröder, A. Strahlberger)
- Symbolic algorithm for solving Emerson-Lei games (Mathieu, Nir)
- Reducing fair games to standard games (I. Saglam, A. Schmuck, Nir)
- Accelerated solution for tree-parts of parity games
- Faster and smaller solution for obliging games (Nir)

Emerson-Lei Games

Emerson-Lei Games

$$G = (V, E \subseteq V \times V, \mathsf{col} : V \to 2^{\mathsf{C}}, \varphi) \qquad \varphi \in \mathbb{B}(\mathsf{GF}(\mathsf{C}))$$

Player $\exists$ wins play π iff $\mathsf{col}[\pi] \models \varphi$

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Examples:

$C = \{f\}$	$\varphi = \mathrm{GF}\, f$	(Büchi)
$C = \{f_1, \ldots, f_k\}$	$\varphi = \bigwedge_{1 \le i \le k} \mathrm{GF}\, f_i$	(gen. Büchi)
$C = \{p_1, \ldots, p_{2k}\}$	$\varphi = \bigvee_{i \text{ even}} \mathrm{GF}\, p_i \wedge \bigwedge_{j>i} \mathrm{FG} \neg p_j$	(parity)
$C = \{e_1, f_1, \ldots, e_k, f_k\}$	$\varphi = \bigvee_{1 \le i \le k} \mathrm{GF}\, e_i \wedge \mathrm{FG} \neg f_i$	(Rabin)
$C = \{r_1, g_1, \ldots, r_k, g_k\}$	$\varphi = \bigwedge_{1 \le i \le k} \mathrm{GF}\, r_i \to \mathrm{GF}\, g_i$	(Streett)

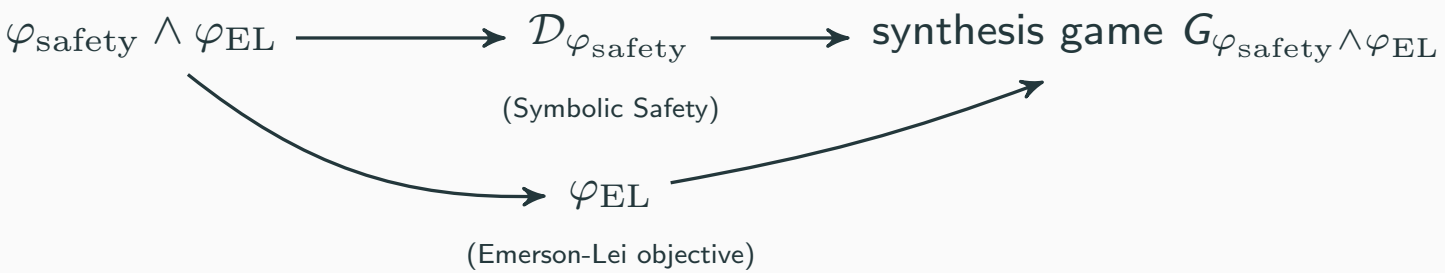
Determined, not positional (in general: memory $|C|!$)

Emerson-Lei Games, results

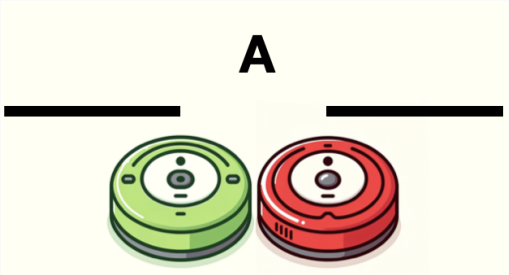
Main results:

- Direct fixpoint characterization of Zielonka trees
- Adaptive symbolic fixpoint algorithm for Emerson-Lei games
- Solves Emerson-Lei games with n nodes, k colors in time $\mathcal{O}(k! n^{\frac{k}{2}})$

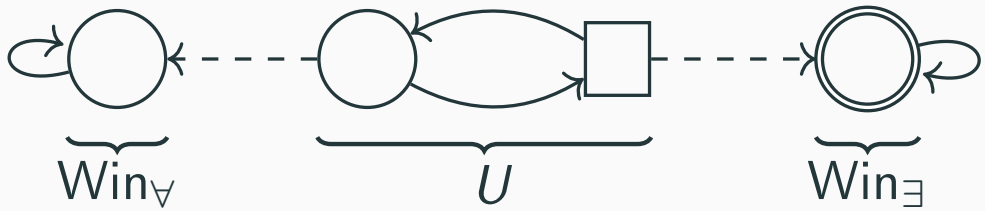
Application: Symbolic reactive synthesis for EL+safety fragment of LTL



Fair Games



- Both green robot and red robot want to reach A first
- Deadlock if neither gives way by moving aside
- The one moving aside first loses



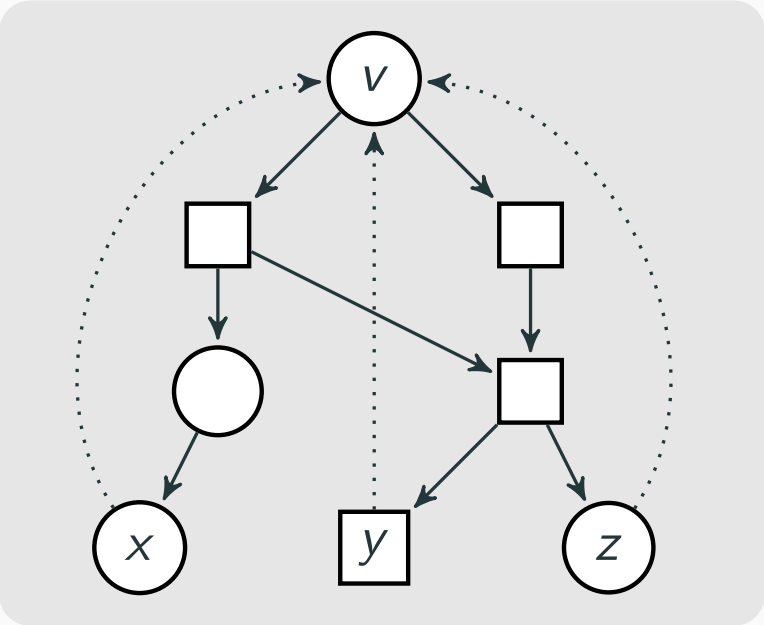
$\rightsquigarrow$ Introduce the notion of fair α/β games, show their determinacy

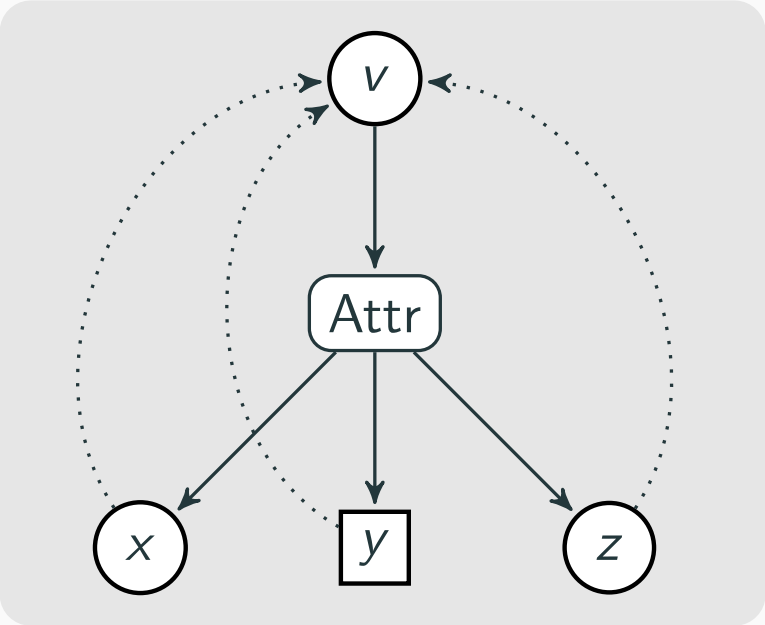
Fair Games, results

- Reduction of fair parity(k)/$\perp$ games to standard parity games of size $k \cdot n$
- Reduction of fair parity(k)/parity(k') games to standard parity games of size $k \cdot k' \cdot n$
- Symbolic algorithm to solve both cases directly

Accelerated Solution of Tree-Parts in Games

n nodes, m cycle-free nodes





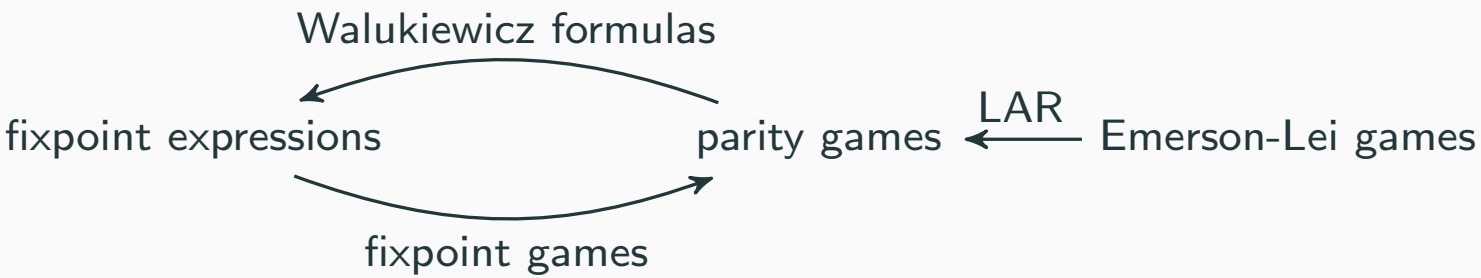
$\nu X.\,\mathrm{Cpre}(X)$

n iterations of Cpre

$\nu Y.\,\mathrm{Attr}(Y)$

$n - m$ iterations of Attr

Accelerated Solution of Tree-Parts in Games, results



- Adapt Walukiewicz formulas to use multi-step attraction (Attr) in place of one-step attraction (Cpre)
- Reduces domain of fixpoint computations $\leadsto$ faster game solving
- Show that LAR reduction preserves tree-like sub-games

Summary

Take-away:

- Games capture central algorithmic content of many problems in FM
- Better game solving algorithms / smarter game reductions lead to improved problem solving

Ongoing work:

- Faster and Smaller Solution of Obliging Games