Emerson-Lei Games

Recent work on improved game analysis

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Emerson-Lei Objectives

Emerson-Lei Games

$$G = (V, E \subseteq V \times V, \mathsf{col} : V \to 2^{\mathsf{C}}, \varphi) \qquad \varphi \in \mathbb{B}(\mathsf{GF}(\mathsf{C}))$$

Player $\exists$ wins play π iff $\mathsf{col}[\mathsf{Inf}(\pi)] \models \varphi$

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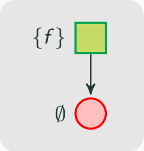
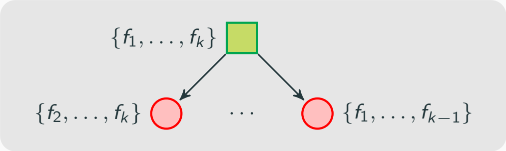
Examples:

$C = \{f\}$	$\varphi = \mathsf{GF}\, f$	(Büchi)
$C = \{f_1, \ldots, f_k\}$	$\varphi = \bigwedge_{1 \leq i \leq k} \mathsf{GF}\, f_i$	(gen. Büchi)
$C = \{p_1, \ldots, p_{2k}\}$	$\varphi = \bigvee_{i \text{ even}} \mathsf{GF}\, p_i \wedge \bigwedge_{j>i} \mathsf{FG}\, \neg p_j$	(parity)
$C = \{e_1, f_1, \ldots, e_k, f_k\}$	$\varphi = \bigvee_{1 \leq i \leq k} \mathsf{GF}\, e_i \wedge \mathsf{FG}\, \neg f_i$	(Rabin)
$C = \{r_1, g_1, \ldots, r_k, g_k\}$	$\varphi = \bigwedge_{1 \leq i \leq k} \mathsf{GF}\, r_i \to \mathsf{GF}\, g_i$	(Streett)

Zielonka Trees by Example



Büchi objective



generalized Büchi objective

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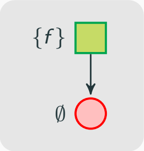

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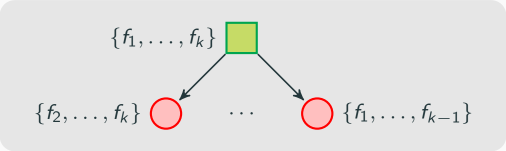

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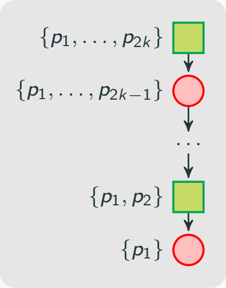
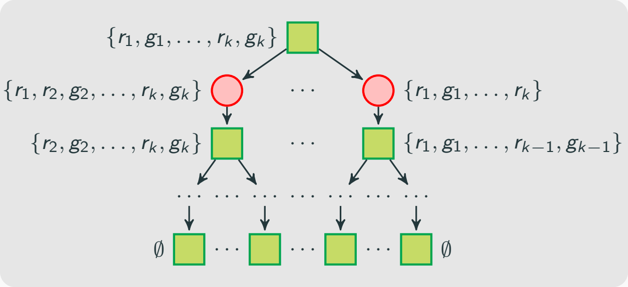
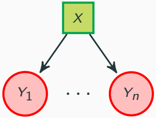


parity objective

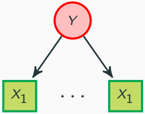


Streett objective

Fixpoint Systems from Zielonka Trees



$$\rightsquigarrow X =_{\nu} Y_1 \cap \ldots \cap Y_n$$



$$\rightsquigarrow Y =_{\mu} X_1 \cup \ldots \cup X_n$$

$$\rightsquigarrow Z =_{\nu} \bigcup_{Z_i \geq Z} \mathsf{col}^{-1}_{Z,Z_i} \cap \mathsf{Cpre}(Z_i)$$

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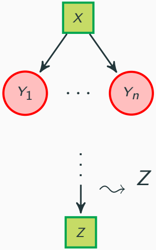

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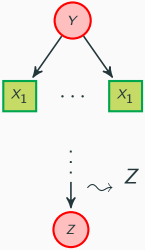

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Instantiates to known characterizations in all mentioned examples, e.g.

$$\nu X.\, \mu Y.\, (f \cap \mathrm{Cpre}(X)) \cup \mathrm{Cpre}(Y) \qquad \text{(Büchi)}$$

$$\nu X_{2k}.\, \mu X_{2k-1}.\, \ldots\, .\mu X_1.\, \bigcup_{1 \leq i \leq 2k} p_i \cap \mathrm{Cpre}(X_i) \qquad \text{(parity)}$$

Universal Trees

Recent progress:

- (Calude et al.): Solve parity games with universal trees of QP size
- (H, Schröder, 2021): Solve fixpoint systems using universal trees

Generalizes to many-valued energy games: $\mathsf{Cpre}_q : c^V \to c^V$

stochastic games: $\mathsf{Cpre}_s : [0,1]^V \to [0,1]^V$

Ongoing work:

- Tight bounds on universal tree size beyond parity objectives (such as Rabin, Streett or Emerson-Lei objectives); conjecture: $\mathcal{O}(k!n^{\log k})$

Summary

Take-away:

- Adaptive fixpoint algorithm for Emerson-Lei games via Zielonka trees
- Solves Emerson-Lei games with k colors, n nodes in time $\mathcal{O}(k!n^k)$

Other topics:

- Polynomial determinization for 2-bounded Streett automata
- Reducing fair parity games to parity games, generalizing reduction from stochastic parity games to parity games